day 37

HW/ 3.7/59-64, 69, 75, 79-80
3.8/9-10, 13-16, 25, 31-32

3.4/83 we suspect e^{Kx} is involved somehow...

Let $f(x) = e^{Kx}$ and $g(x) = e^{Lx}$

$f'(x) = Ke^{Kx}$; $g'(x) = Le^{Lx}$.

$\frac{d}{dx}(f \cdot g) = \frac{d}{dx}\left(e^{Kx} \cdot e^{Lx}\right) = \frac{d}{dx}\left(e^{(K+L)x}\right) = (K+L)e^{(K+L)x}$

$f'(x) \cdot g'(x) = KLe^{(K+L)x}$

so we need $K+L = K \cdot L$

$K = KL - L = L(K-1)$

$\frac{K}{K-1} = L$

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3.8/5

day 37

$x^4+y^4=2$ [find slope at $(1,-1)$]

implicit diff

$\frac{d}{dx}f(x) \Rightarrow$ normal

$\frac{d}{dx}g(y)=g'(y)\cdot\frac{dy}{dx}$

[chain rule]

deriv
solve for $\frac{dy}{dx}$:

$$\frac{d}{dx}(x^4)+\frac{d}{dx}(y^4)=\frac{d}{dx}(2)$$

$$4x^3+4y^3\cdot\frac{dy}{dx}=0$$

$$-4x^3 \qquad -4x^3$$

$$\frac{4y^3}{4y^3}\frac{dy}{dx}=\frac{-4x^3}{4y^3}$$

$$\frac{dy}{dx}=\frac{-4x^3}{4y^3}=-\frac{x^3}{y^3}$$

$$\frac{d}{dx}\left([f(x)]^4\right)=4[f(x)]^3\cdot f'(x)$$

$$\left.\frac{dy}{dx}\right|_{(1,-1)}=-\frac{(1)^3}{(-1)^3}=1$$

day 37

$$x^4 + y^4 = 2$$
$$-x^4 \qquad -x^4$$
$$y^4 = 2 - x^4$$
$$y = \begin{cases} +\sqrt[4]{2-x^4} \\ -\sqrt[4]{2-x^4} \end{cases}$$

$$y = \sqrt[4]{2-x^4} = (2-x^4)^{\frac{1}{4}}$$
$$y' = \frac{1}{4}(2-x^4)^{-\frac{3}{4}} \frac{d}{dx}(2-x^4) = \frac{1}{4}(-4x^3)(2-x^4)^{-\frac{3}{4}}$$
$$= \frac{-x^3}{\sqrt[4]{(2-x^4)^3}}$$

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day 37

$(37)^3$ Let $h(x) = f(g(x))$; $p(x) = g(f(x))$

a) $h'(3) = f'(g(3)) \cdot g'(3) = f'(1) \cdot (20) = (5) \cdot (20) = 100$

1) find $h'(x)$

2) replace x with 3

$h'(x) = \frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

CHAIN RULE

b) $h'(2) = f'(g(2)) \cdot g'(2) = f'(5) \cdot 10 = (-10)(10) = -100$

c) $p'(4) = p'(x)\Big|_{x=4}$

$= \frac{d}{dx}(g(f(x)))\Big|_{x=4} = g'(f(x)) \cdot f'(x)\Big|_{x=4}$

$= g'(f(4)) \cdot f'(4) = g'(1)(-8) =$

$= (2)(-8) = -16$

$y = f(x)$

$\frac{dy}{dx}$; y' ; $f'(x)$

$\frac{d}{dx}(?)$

day 37

3.7/3.19

$$y = \left(\frac{x}{x+1}\right)^5$$

$$x \xmapsto{\frac{x}{x+1}} \frac{x}{x+1} \xmapsto{x^5} \left(\frac{x}{x+1}\right)^5$$

$$y' = 5\left(\frac{x}{x+1}\right)^4 \cdot \frac{d}{dx}\left(\frac{x}{x+1}\right)$$

$$= 5\left(\frac{x}{x+1}\right)^4 \left(\frac{\frac{d}{dx}(x)(x+1) - x\cdot\frac{d}{dx}(x+1)}{(x+1)^2}\right)$$

$$= 5\left(\frac{x}{x+1}\right)^4 \left(\frac{(x+1)-(x)}{(x+1)^2}\right) = 5\left(\frac{x}{x+1}\right)^4\left(\frac{1}{(x+1)^2}\right)$$

$$= \frac{5x^4}{(x+1)^6}$$

day 37

3.7/2.29

$$y=\left(\frac{e^x}{x+1}\right)^8$$

$$y'=8\left(\frac{e^x}{x+1}\right)^{8-1}\cdot\frac{d}{dx}\left(\frac{e^x}{x+1}\right)$$

$$=8\left(\frac{e^x}{x+1}\right)^7\frac{\underbrace{\frac{d}{dx}(e^x)}_{e^x}(x+1)-e^x\underbrace{\left(\frac{d}{dx}(x+1)\right)}_{1}}{(x+1)^2}$$

$$=8\left(\frac{e^x}{x+1}\right)^7\frac{e^x(x+1)-e^x}{(x+1)^2}=8\left(\frac{e^x}{x+1}\right)^7\left(\frac{xe^x}{(x+1)^2}\right)$$

$$=\frac{xe^{8x}}{(x+1)^9}$$

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day 37

3.9 and exponential & logarithm things * and inverse functions

calculator

slope we have a tangent to C_1 at $(1,e)$

slope and a tangent to C_2 at $(e,1)$

MODE: PAR

$X_1 = T$

$Y_1 = e^T$

graph

draw tangent at $(1,e)$

MORE

$X_2 = e^T$

$y_2 = T$

graph

2ND DRAW TANGENT

$\downarrow X_2 = \quad Y_2 =$

1

draw tan $(e,1)$ at $T=1$

$slope_1 = 2.718$

$slope_2 = .3678$

$2.718 * .3678 = .9996\ldots$